Introduction to Humidity
Basic Principles on Physics of Water Vapor

Preface

This Application Note gives a short introduction to physics of humidity and the most important formulae. It puts relation between the most important entities and provides some physical framework.

Sensirion is a manufacturer of humidity and temperature sensors – see SHTxx product line. Starting from relative humidity and temperature, this Application Note helps to calculate most physical values describing water vapor.

1 Basic Considerations

Humidity describes the quantity of water vapor in a gas like air. There are many different ways to express humidity, e.g. relative humidity, absolute humidity, dew point temperature or mixing ratio. In the following, the origin of water vapor in air is discussed and the most frequently used definitions for humidity are described.

1.1 The Origin of Water Vapor

Consider a closed chamber with liquid water as shown in Figure 1. The average kinetic energy, and thus the impulse of the water particles in the liquid water, is governed by temperature. Some particles have higher energy than the average and can escape from the liquid – so called evaporation above liquid. Since the gaseous particles bounce around the closed chamber some of them will hit the surface of the liquid water and may be trapped there again. After some time, there will be equilibrium where the number of water particles leaving the surface of the liquid water is equal to the number rejoining it. If this equilibrium is achieved, the number of gaseous water particles remains statistically constant.
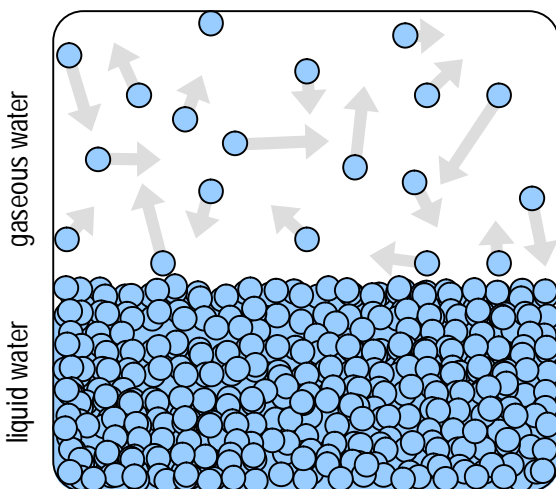
Figure 1 Closed chamber with liquid water and gaseous water particles (vapor). In equilibrium, the number of water particles leaving the surface of the liquid water is equal to the number rejoining it. The gaseous water particles exert a pressure e to the wall.

1.2 Saturated Vapor Pressure e_w and e_i

Since the gaseous water particles hit the wall of the closed chamber in Figure 1, a pressure e (e stands for *evaporation* and is used for partial water pressure only) is exerted to the wall. In equilibrium, this pressure is called saturated vapor pressure e_w above water or e_i above ice, which can be expressed by the Clapeyron relation:

$$\frac{de_w}{dT} = \frac{L_v}{T \Delta V}, \tag{1}$$

where L_v is the latent heat, T the absolute temperature in Kelvin, and ΔV the volume change of the phase transition. Since the specific volume of the liquid is much smaller than that of the gas and assuming the vapor gas to be ideal ($pV = nRT$), equation (1) can be approximated by the Clausius-Clapeyron equation

$$\frac{de_w}{dT} = \frac{L_v \cdot e_w}{R_v \cdot T^2}, \tag{2}$$

where R_v is the water vapor gas constant. Solving differential equation (2) leads to the following equation for the saturated vapor pressure – known as the Magnus formula

$$e_w(t) = \alpha \exp\left(\frac{\beta \cdot t}{\lambda + t}\right) \tag{3}$$

where t is the temperature in °C and the parameters α, β, and λ are given in Table 1.

Condition	T range (°C)	α (hPa)	β	λ (°C)
Above water	-45 – 60	6.112	17.62	243.12
Above ice	-80 – 0.01	6.112	22.46	272.62

Table 1 Magnus Parameters for equation (3).

The saturated vapor pressure above water is plotted in Figure 2. The corresponding saturated vapor pressure above ice $e_i(t)$ is calculated according to equation (3) using the corresponding parameters from Table 1. Between temperatures of -45°C and 60°C, equation (3) has an uncertainty of less than ±0.6% at 95% confidence level. If more accuracy is required, equations for saturated vapor

pressure often lead to implicit functions that can only be solved numerically. The derivation of most formulae in the next sections is based on equation (3).
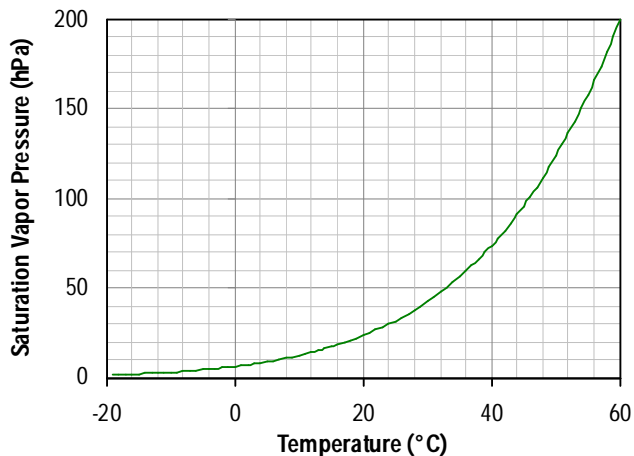
Figure 2 Saturated Vapour Pressure $e_w(t)$

1.3 Water Vapor Enhancement Factor

In air, additional gases or gas mixtures are present. In this case, the total barometric pressure is the sum of the partial pressures

$$p = p_{nitrogen} + p_{oxygen} + p_{others} + e \ . \tag{4}$$

Since the different gas particles in air interact, a pressure dependent correction may be required to adjust the water vapor pressure. The real actual vapor saturation pressure above water e'_w or ice e'_i is given by

$$e'_w = e_w \cdot f_w \quad \text{or} \quad e'_i = e_i \cdot f_i \tag{5}$$

where the water vapor enhancement factor f is given by

$$f_w(p) = 1.0016 + 3.15 \cdot 10^{-6} \cdot p - 0.074 \cdot \frac{1}{p} \tag{6}$$

and the barometric pressure p can range from 3kPa to 110kPa. At normal barometric pressure, it can be assumed that $f = 1$. At 6bar, the enhancement factor is 1.02.

2 Terms of Humidity

2.1 Relative Humidity U_w and U_i

Relative humidity above water U_w or above ice U_i is mostly used for measurement purposes. It is defined as the ratio of the partial vapor pressure in air e to the saturated vapor pressure at a given temperature t, $e_w(t)$

$$U_w(t) = \frac{e}{e_w(t)} \cdot 100\% \tag{7}$$

and above ice

$$U_i(t) = \frac{e}{e_i(t)} \cdot 100\% \ . \tag{8}$$

Therefore, the relative humidity in the closed chamber of Figure 1 is exactly 100%RH when equilibrium is achieved.

The value of relative humidity is very sensitive to temperature. Note, that at relative humidity of about 90%RH at ambient temperature a temperature deviation of $\Delta t = 1°C$ results in a change of up to $\Delta U \approx -5\%$RH.

Calculation of relative humidity at different temperature: Consider a quantity of air with a constant number of gas and water molecules with relative humidity U_1 at temperature t_1. If, at constant pressure, the temperature is changed from t_1 to t_2, the corresponding relative humidity U_2 gets

$$U_2 = U_1 \cdot \frac{\exp\left(\frac{\beta \cdot t_1}{\lambda + t_1}\right)}{\exp\left(\frac{\beta \cdot t_2}{\lambda + t_2}\right)} \ . \tag{10}$$

It is assumed that no condensation or icing occurs during or after the temperature change from t_1 to t_2.

Conversion from relative humidity measured above water U_w to relative humidity measured above ice U_i:

$$U_i = U_w \cdot \frac{\exp\left(\frac{\beta_w \cdot t}{\lambda_w + t}\right)}{\exp\left(\frac{\beta_i \cdot t}{\lambda_i + t}\right)} \ . \tag{11}$$

Most humidity sensors are calibrated relative to $e_w(t)$, even below 0°C (super cooled water) according to World Meteorological Organization (WMO). For conversion to values relative to $e_i(t)$ equation (11) should be used.

2.2 Dew Point Temperature t_d

Consider a quantity of air with constant number of water particles (i.e. no condensation or evaporation) at a certain temperature t and relative humidity $U < 100\%$. The *dew point temperature t_d* is defined as the temperature to which this quantity of air must be cooled down such that, at constant pressure, condensation occurs ($U = 100\%$). It can be derived by using equation (10) with $U_1 = 100\%$ and $t_1 = t_d$ and then resolve it to t_d:

$$t_d(t, U_w) = \lambda \frac{\ln\left(\frac{U_w}{100\%}\right) + \frac{\beta \cdot t}{\lambda + t}}{\beta - \ln\left(\frac{U_w}{100\%}\right) - \frac{\beta \cdot t}{\lambda + t}} \ . \tag{12}$$

For calculating frost temperature t_f, where ice is desublimated out of saturated air, the same formula is

applied – with U_w replaced by U_i and Magnus coefficients α, β and λ defined above ice (see Table 1).

2.3 Absolute Humidity d_v

Absolute humidity is defined by the mass of water vapor m_{H2O} per humid air volume V and can be expressed as

$$d_v = \frac{m_{H_2O}}{V} \ . \tag{13}$$

Applying the ideal gas law to partial water pressure e, $eV = nRT$, where V is the volume, n the number of water particles in mol, $R = 8.314472 \ J \cdot mol^{-1} \cdot K^{-1}$ the universal gas constant and T the absolute temperature, and applying the molar weight of water $M_{H2O} = 18.0 g/mol$ the following equation results

$$m_{H_2O} = M_{H_2O} \cdot n = 18.0 \cdot n = 18.0 \cdot \frac{e \cdot V}{RT} \ . \tag{14}$$

Combining equation (13) and equation (14), one derives

$$d_v = \frac{m_{H_2O}}{V} = 18.015 \cdot \frac{e}{RT} = 2.1667 \cdot \frac{e}{T} \tag{15}$$

with T in Kelvin, e in Pa and d_v in g/m^3. Absolute humidity in g/m^3 with e in hPa and t in °C is given by

$$d_v = 216.7 \cdot \frac{e}{(273.15°C + t)} \tag{16}$$

or by substituting e from equation (3)

$$d_v = 216.7 \cdot \frac{\frac{U_w}{100\%} \cdot \alpha \cdot \exp\left(\frac{\beta \cdot t}{\lambda + t}\right)}{(273.15°C + t)} \ . \tag{17}$$

2.4 Mixing Ratio r

Mixing Ratio is the ratio of the mass of water vapor and the mass of dry air. Using the molar mass of dry air and water vapor

$$M_{dryair} = 29.0 \ g \cdot mol^{-1}$$

$$M_{H_2O} = 18.0 \ g \cdot mol^{-1}$$

one derives

$$r = \frac{M_{H_2O}}{M_{dryair}} = \frac{18.0 \cdot \frac{e}{R \cdot T}}{29.0 \cdot \frac{p - e}{R \cdot T}} = 0.622 \cdot \frac{e}{p - e} \tag{18}$$

in g/g with the total barometric pressure p. Mixing ratio r in $(g \cdot kg^{-1})$ is expressed in terms of relative humidity and temperature using equation (3):

$$r = \frac{622 \cdot e}{p - e} = 622 \cdot \frac{\alpha \cdot \frac{U_w}{100\%} \cdot \exp\left(\frac{\beta \cdot t}{\lambda + t}\right)}{p - \alpha \cdot \frac{U_w}{100\%} \cdot \exp\left(\frac{\beta \cdot t}{\lambda + t}\right)} \ . \tag{19}$$

2.5 Heat Index HI

Heat Index indicates how the human body feels temperature. If relative humidity is low human body cools itself by perspiration, dissipating heat from the body. At higher relative humidity the evaporation rate from the human skin is lower. In that case, the body cannot dissipate heat as easily as it is the case in dry air.

The heat index is based on subjective measurements and is only meaningful above 25°C and 40%RH. The most popular definition of the Heat Index is the one of the *National Weather Service and Weather Forecast Office* of the *National Oceanic and Atmospheric Administration* (NOAA). The Heat Index in °C is given by
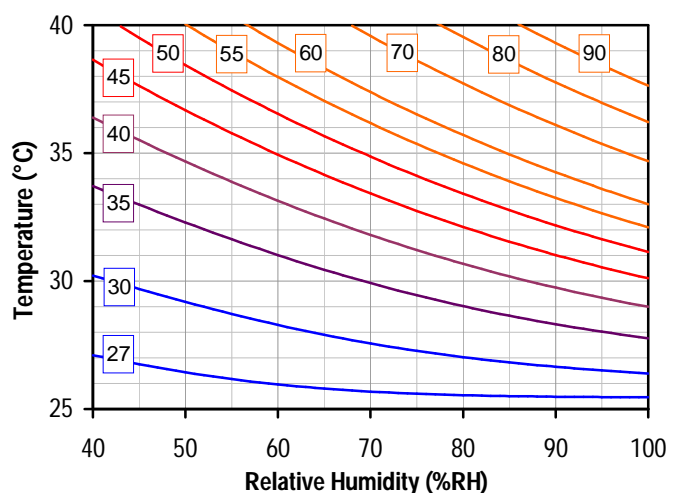
$$HI = c_{00} + c_{10}t + c_{01}U_w + c_{11}tU_w + c_{20}t^2 + c_{02}U_w^2 \\ + c_{21}t^2U_w + c_{12}tU_w^2 + c_{22}t^2U_w^2 \tag{20}$$

with the following coefficients:

c_{00}	-8.7847	c_{02}	-0.0164
c_{10}	1.6114	c_{21}	$2.2117 \cdot 10^{-3}$
c_{01}	2.3385	c_{12}	$7.2546 \cdot 10^{-4}$
c_{11}	-0.1461	c_{22}	$-3.5820 \cdot 10^{-6}$
c_{20}	-0.0123		

Table 2 Coefficients for Heat Index formula

Figure 3 displays the Heat Index as a function of relative humidity and temperature.

Figure 3 Heat Index in °C. Values below 30°C: no discomfort, 30 – 40°C: some discomfort, 40 – 45°C: great discomfort, above 45°C: dangerous, above 54°C: heat stroke imminent.

3 Properties of Humid Air

3.1 Specific Enthalpy

Specific enthalpy h in kJ/kg is the sum of energies needed to warm up 1kg dry air from 0°C to t, evaporating vapor inside moist air and warming up the vapor from 0°C to t:

$$h = c_{pa} t + \left(l_w + c_{pv} t \right) \cdot r \tag{21}$$

with the heat capacity of air c_{pa} = 1.005 kJ/(kg·K), the heat capacity of water vapor c_{pv} = 1.859 kJ/(kg·K) and the latent heat of water vapor l_w = 2501 kJ/kg. r is the mixing ratio introduced by equation (18).

3.2 Density of Humid Air

The density ratio between dry and humid air is given by

$$\frac{\rho_h}{\rho_{air}} = \left(\frac{U_w \cdot \alpha}{100\% \cdot p} \cdot \exp\left(\frac{\beta \cdot t}{\lambda + t} \right) \right) \cdot \left(\frac{18.0}{29.0} - 1 \right) + 1 \tag{22}$$

where ρ_h is the density of humid air, ρ_{air} the density of dry air and p the barometric pressure in Pa. The parameters α, β and λ are given in Table 1. Analyzing the equation it becomes clear that humid air has a lower density than dry air.

3.3 Heat Capacity of Humid Air

Heat capacity of humid air can be expressed as

$$\frac{\rho_h \cdot C_{Ph}}{\rho_{air} \cdot C_{Pair}} = \left(\frac{U_w \cdot \alpha}{p} \cdot \exp\left(\frac{\beta \cdot t}{\lambda + t} \right) \right) \cdot \left(\frac{33.6}{29.1} - 1 \right) + 1 \tag{23}$$

with heat capacity of humid air C_{Ph} and dry air C_{Pair}. The equation reveals that the heat capacity of humid air is higher than that of dry air.

4 Applications

## 4.1 Psychrometric Chart - Mollier Diagram

Psychrometric charts or Mollier diagrams display aequi-potential curves of enthalpy, relating relative humidity on charts of mixing ratio and temperature. They help to navigate in systems where humidity and temperature play an important role – e.g. in a tumble dryer.

A typical Mollier diagram is shown in Figure 4. Aequi-potential curves of enthalpy (blue lines) are derived from equation (21) while the curves of equal relative humidity (green lines) are derived from equation (19).

As an example let us consider a tumble dryer for laundry drying: In a first step a quantity of air with fix mixing ratio is heated up outside the tumbler drum – represented as a horizontal path on the chart to the right. In a second step the same quantity of air is lead to the drum where the air absorbs water vapor from the wet laundry – the path on

the chart goes along a blue energy aequi-potential line as no energy is added at that step. Eventually the wet air is extracted from the drum and cooled down to the initial temperature – the path goes to the right with fix mixing ratio until it hits dew point temperature and then follows the 100%RH line.
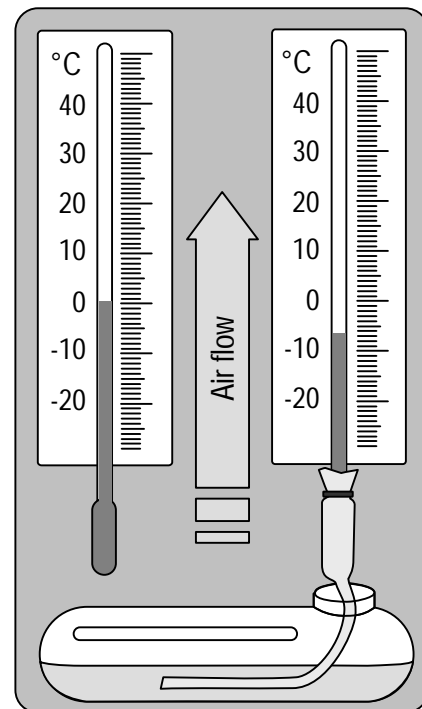
For a temperature cycle between 10°C and 40°C, energy of 30kJ per kg air is consumed and up to 8g water per kg air and drying cycle may be condensed.
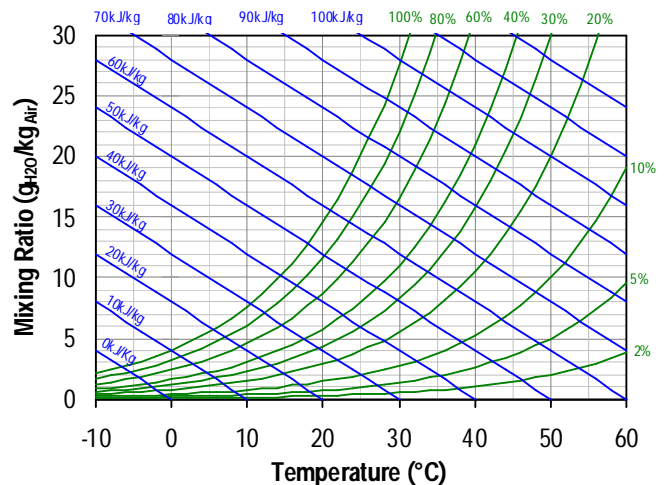
Figure 4 Mollier Diagram: Blue lines are of equal enthalpy, red lines denote equal relative humidity.

4.2 Wet- and Dry-Bulb Temperature

A wet- and dry-bulb hygrometer is shown in Figure 5.

Figure 5 Sketch of a wet bulb hygrometer or psychrometer. The wet bulb thermometer measures lower temperature than the dry bulb due to evaporation at its bulb – evaporation that at its side depends on the local relative humidity.

Two thermometers, of which one of them is is enclosed in a porous medium which is maintained wet by a wet sock or wick measure temperatures t_{DB} (dry bulb temperature) and t_{WB} (wet bulb temperature). As the wet sock evaporates water its temperature is reduced – and as the evaporation is stronger in dry air than in wet air, the temperature difference is a measure for relative humidity.

This wet and dry bulb hygrometer is also called psychrometer – *psychros* means "cold" in Greek language. Humidity can be calculated by using the psychrometer equation

$$e = e_w(t_{DB}) - B \cdot p \cdot (t_{DB} - t_{WB}) \tag{25}$$

where p is the total barometric pressure in hPa and B the psychrometer coefficient that varies between $B = 6.4 \cdot 10^{-4}$ °C^{-1} and $B = 6.8 \cdot 10^{-4}$ °C^{-1} depending on the ventilation of the psychrometer. Individual values are assigned by calibration of the psychrometer.

Relative humidity can be calculated by substituting equation (25) with equations (3) and equation (7)

$$U_w(t_{DB}, t_{WB}) = 100\% \cdot \frac{\alpha \cdot \exp\left(\dfrac{\beta \cdot t_{DB}}{\lambda + t_1}\right) - B \cdot p \cdot (t_{DB} - t_{WB})}{\alpha \cdot \exp\left(\dfrac{\beta \cdot t_{DB}}{\lambda + t_{DB}}\right)} \ . \tag{26}$$

Resolving equation (26) on t_{WB} allows for expressing wet bulb temperature in function of relative humidity U_w and temperature t:

$$t_{WB} = t \ - \frac{\alpha}{B \cdot p} \cdot \exp\left(\frac{\beta \cdot t}{\lambda + t}\right) \cdot \left(1 - \frac{U_w}{100\%}\right). \tag{27}$$

5 Glossary

The following Table gives symbols according to VDI/VDE 3514 standard.

Symbol	Physical Value	Unit
U	Relative humidity	%RH
U_w	Relative humidity above water	%RH
U_i	Relative humidity above ice	%RH
p	Barometric pressure	hPa
$p_{nitrogen}$	Partial nitrogen pressure (analogue for other gases, except water)	hPa
e	Partial water pressure	hPa
e_w	Saturated vapor pressure above water	hPa
e'_w	Saturated vapor pressure above water in real gas mixture	hPa
e_i	Saturated vapor pressure above ice	hPa
e'_i	Saturated vapor pressure above ice in real gas mixture	hPa
t, T	Temperature	°C, K
t_d	Dew point temperature	°C
t_f	Frost temperature	°C
r	Mixing ratio	g/kg
d_v	Absolute humidity	g/m^3
h	Specific enthalpy	J/kg

Revision History

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